TABLE 1. REPEPTIZATION RATES AND EQUILIBRIA[a]

Ionic strength (mol/l)	$\frac{\phi_{mn2}}{kT}$	$\frac{\phi_{max}}{kT}$	K_r' (cm²/sec)	K_f' (cm³/sec)	τ (sec)	c_∞^* (cm⁻³)
0.08	−21.4	2.93	6.1×10^{-10}	2.6×10^{-9}	3.8	6.8×10^{8}
0.07	−18.2	5.26	1.7×10^{-9}	2.8×10^{-10}	36	1.8×10^{10}
0.06	−13.8	8.55	5.1×10^{-9}	1.1×10^{-11}	900	1.3×10^{12}

[a] Values of parameters used in these calculations include $a = 10^{-5}$ cm, $a_f = 10^{-3}$ cm, $c_f = 10^{8}$ cm^{-3}, $\sigma = 2.5$A, $\epsilon = 74.3$, $T = 300$°K, $\psi = -10$mV at 0.1 mol/l (recomputed from Gouy-Chapman equation at lower ionic strengths, assuming surface charge is unchanged) and $A = 1.769 \times 10^{-14}$ erg (chosen so that $\phi_{max} = 0$ at 0.1 mol/l). *For Notations see Ruckenstein and Prieve (1976).*

cles in flocs equals a constant, c_o, per unit volume. Equation (D) can be integrated exactly, taking into account the changes of K_r' and K_f' with time. However, more insight into the problem can be obtained from the simple equation obtained in the limiting case when the amount of dilution of the electrolyte is small so that nearly all the elementary particles remain in the flocs. Then K_r' and K_f', which are proportional to a_f^2, remain nearly constant. Interpreting n as the number of elementary particles attached per unit area to the floc and assuming hexagonal close packing, then $n = (\sqrt{3})/6a^2$. As particles detach from the floc, new particles underneath are exposed so that n is practically independent of time. Then (D) integrates to yield

$$(c_\infty^* - c_\infty(t)) = (c_\infty^* - c_{\infty i})e^{-t/\tau}$$

where $c_\infty^* = K_r' n / K_f'$, $\tau = 1/K_f' c_f$ and $c_{\infty i} = c_\infty(t = 0)$.

Table 1 shows some values of the time constant τ and the equilibrium concentration c_∞^* obtained from these equations, employing the simplified model of the interaction forces developed by Ruckenstein and Prieve (1976). As might be expected, lowering the ionic strength makes the primary minimum, ϕ_{mn2}, less deep, thus shifting the equilibrium in favor of more detached elementary particles, c_∞^*. However, decreasing the ionic strength increases the time τ to reach equilibrium, in spite of the fact that the repeptization rate, $K_r'n$, is increasing. This can be rationalized by observing that both the repeptization rate $K_r' n$ and the equilibrium concentration c_∞^* are increased by dilution, but c_∞^* increases faster so that a longer time τ is needed to reach equilibrium. Of course, the analysis here could be generalized to allow the recombination of singlets to form doublets, the combination of singlets and doublets to form triplets and quadruplets, etc. Finally, an equilibrium distribution of floc sizes should be obtainable.

In this note, an equation is established for the rate of reversible adsorption on the basis of the quasi steady state assumption over the entire thickness of the interaction force boundary layer. When an interaction force boundary layer occurs ($\phi_{max} >> kT$), we find that the approximation of quasi steady state over part of the region and quasi-equilibrium over the other part leads to practically the same rate as the approximation of quasi steady state over the entire region if $\phi_{max} - \phi_{mn2} >> kT$. Hence, either approximation is equally appropriate. A simple analysis of the repeptization of colloids is discussed on the basis of the equation established for reversible adsorption.

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High Peclet Number Mass Transfer to a Sphere in a Fixed or Fluidized Bed

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The mass transfer from a moving fluid to a solid particle situated in a fixed or fluidized bed is encountered in a large number of industrial processes (chemical reactors, filters, etc.). Practically important and very attractive is the case of low Reynolds number (creeping) flow of the fluid through the bed. In this case, an analytical solution is possible. It is based on Levich's (1962) well-known solution for the mass transfer to a single sphere falling in an infinite fluid with velocity U_o. His solution applies for the case where the diffusion coefficient of the species D is low; in other words, the characteristic Peclet number Pe of the process is high ($Pe \geqq 1\,000$). As pointed out by several authors, this requirement is met in most practical cases. In this situation, the mass transfer occurs in a very thin concentration boundary layer near the solid sphere, for which the spherical geometry can be neglected. Mass transfer to a single sphere can be expressed in terms of the Sherwood number as

$$Sh = 0.997 \cdot Pe^{1/3} \tag{1}$$

Equation (1) was also obtained by Friedlander (1961) and Lochiel and Calderbank (1964), while a second-order correction to this expression was introduced by Acrivos and Goddard (1965, 1966).

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The case of diffusion controlled fine dust filtration by spherical filter elements may be handled identically to that of mass transfer to spheres. There, however, it is customary to express the results in terms of collection efficiencies (Tardos et al., 1976). Thus Equation (1) may be rewritten as

$$E = Sh \frac{4}{Pe} = 3.988 \cdot Pe^{-2/3} \tag{2}$$

In the present note, we derive a general expression for the mass transfer to a sphere inside a granular bed in the form of a correction factor to Equation (1), which is a function of the velocity profile around the sphere. We then demonstrate its use by applying it to two different velocity profiles. The resultant mass transfer rates are compared to other theoretical and experimental results from the literature.

The diffusion equation that applies for the present case is

$$U_r \frac{\partial N}{\partial R} + \frac{V_\theta}{R} \frac{\partial N}{\partial \theta} = \frac{D}{U_o a} \frac{\partial^2 N}{\partial R^2} \tag{3}$$

where U_r and V_θ are the dimensionless velocity components of the flow around a sphere. Levich in his solution took those from the stream function ψ for creeping flow (Lamb, 1932):

$$\psi = -\frac{1}{4}\left(2R^2 - 3R + \frac{1}{R}\right)\sin^2\theta \tag{4}$$

Then, application of von Mises' transformation to Equation (3) simplified the problem to the solution of

$$\frac{\partial N}{\partial \theta} = \frac{2}{Pe} \sin^2\theta \frac{\partial}{\partial \psi}\left(V_\theta \frac{\partial N}{\partial \psi}\right) \tag{5}$$

For a thin concentration boundary layer, one may substitute for the radial coordinate

$$R = 1 + y$$

where y is a small distance. Thus, the stream function from (4) can be rewritten approximately as

$$\psi \cong -\frac{3y^2 \sin^2\theta}{4} + \text{terms in } y^3 \text{ and higher order} \tag{6}$$

From this expression for the stream function, Equation (5) was solved by Levich (1962) and the Sherwood number computed from

$$Sh = \int_o^\pi \frac{\partial N}{\partial R}\bigg|_{R=1} \sin\theta \, d\theta$$

resulting in Equation (1).

For a sphere situated in a swarm of similar particles, as may be the case in a fixed or fluidized bed, a correction factor $g(\epsilon)$ must be introduced in Equation (1) which will account for the porosity (ϵ) of the swarm:

$$Sh = 0.997 \cdot g(\epsilon) \cdot Pe^{1/3} \tag{7}$$

The correction factor $g(\epsilon)$ represents the deviation of a given solution from that of Levich, accounting for the difference in the velocity profile. Any new solution consists essentially of solving Equation (5), with a different stream function following closely Levich's steps. Doing this one can easily show that the expression for the correction factor $g(\epsilon)$ is given by

$$g(\epsilon) = [\psi_2/\psi_1]^{1/3} = -\left[\frac{4\psi_2}{3y^2 \sin^2\theta}\right]^{1/3} \tag{8}$$

where ψ_2/ψ_1 is the ratio of the new stream function to that of Levich, Equation (6), both expressed in terms of the coordinate y.

Many models for the flow field in an assemblage of spherical particles were proposed. The best known of those is the so-called free surface model (Happel, 1958), which considers the fluid in the bed divided into cells. Each cell is composed of a sphere surrounded by a concentric volume of fluid. By using this representation, the stream function and velocity profile can be computed, provided certain boundary conditions are postulated on the cell envelope. Happel (1958) considered zero shear as a suitable boundary condition. Using the flow field obtained from these considerations and Levich's approach, Pfeffer (1964) obtained a solution for this case. His solution may be expressed in terms of the correction factor $g(\epsilon)$ as

$$g(\epsilon)_P = \left[\frac{2[1-(1-\epsilon)^{5/3}]}{2 - 3(1-\epsilon)^{1/3} + 3(1-\epsilon)^{5/3} - 2(1-\epsilon)^2}\right]^{1/3} \tag{9}$$

Kuwabara (1959) modified Happel's cell model postulating zero vorticity on the cell boundary. Rewriting his stream function in terms of the coordinate y and using Equation (8), we come up with the following correction factor:

$$g(\epsilon)_K = \left\{\frac{\epsilon}{2 - \epsilon - \frac{9}{5}(1-\epsilon)^{1/3} - \frac{1}{5}(1-\epsilon)^2}\right\}^{1/3} \tag{10}$$

The drawback in using the Happel-Kuwabara approach for computing the flow field is in the fact that the boundary condition on the cell envelope must be postulated in order to obtain a solution. This makes measurements mandatory if one is to decide which of the models approximates reality best.

In the same context, the solution of Sirkar (1974, 1975) must be considered. Here, a different approach to the flow pattern in the swarm of spheres was used, namely, the creeping flow solution of the Navier-Stokes equations for a random cloud of particles, as obtained recently by Tam (1969). Tam's solution should be interpreted as the most probable flow field around one sphere in a swarm. The correction factor for this case as obtained by Sirkar is

$$g(\epsilon)_S = \left[\frac{2 + 1.5(1-\epsilon) + 1.5[8(1-\epsilon) - 3(1-\epsilon)^2]^{1/2}}{\epsilon\,[2 - 3(1-\epsilon)]}\right]^{1/3} \tag{11}$$

This solution breaks down for $\epsilon < 0.33$.

In the present paper, the flow field in a bed of spheres, as proposed by Neale and Nader (1974), will be used in order to compute the correction factor $g(\epsilon)$ in relation (8). This flow field is basically a cell type of model. The test sphere is considered to be surrounded by a spherical fluid envelope whose dimensionless radius l is computed in the same way as in the Happel-Kuwabara model

$$l = \frac{b}{a} = \sqrt[3]{\frac{1}{1-\epsilon}} \tag{12}$$

with a modification that considers the entire sphere swarm as one large exterior porous mass. The equations of creeping flow are solved for the cell region and Darcy's law for the porous matrix:

$$\Delta^2(\Delta^2\psi) = 0 \qquad 1 \leq R \leq l \tag{13}$$

$$-\frac{1}{K}\Delta^2(\bar{\psi}) + \Delta^2(\Delta^2\bar{\psi}) = 0 \quad R \geq l$$

By matching the boundary conditions for these two equations at the cell envelope ($R = l$), the flow pattern in the whole bed is obtained in the form of inner and outer stream functions:

$$\psi = -\frac{1}{2}\left(\frac{A}{\alpha^3}\frac{1}{R} + \frac{B}{\alpha}R + CR^2 + F\alpha^2R^4\right)\sin^2\theta; \quad 1 \leq R \leq l$$

$$\bar{\psi} = -\frac{1}{2}\left[\frac{H}{\alpha^3}\frac{1}{R} + R^2 + \frac{G}{\alpha^2}e^{-R\alpha}\left(1 + \frac{1}{R\alpha}\right)\right]\sin^2\theta; \quad R \geq l \tag{14}$$

where α is a complicated function of the porosity ϵ, given in Table 1 (after Neale and Nader), and the constants A, B, C, F, H, G are functions of α and $\alpha l = \beta$.

Making now the assumption of the thin concentration boundary layer and again letting $R = 1 + y$, we can now write the stream function of the flow near the test sphere approximately as

$$\psi \cong -\frac{3y^2 \sin^2\theta}{4} \cdot \frac{6f}{J} + \text{terms in } y^3 \text{ and higher order} \tag{15}$$

where f and J are functions of α and β given below. Using Equation (8), we obtain the correction factor for this case:

$$g(\epsilon)_N = \left(\frac{6f}{J}\right)^{1/3} = \left\{\frac{6\,[-4\beta^6 - 14\beta^5 - 30\beta^4 - 30\beta^3 + 10\beta^4\alpha^2 - 5\beta^5\alpha^5 + 10\beta^3\alpha^2 + 5\beta^2\alpha^3 - \beta\alpha^5 - \alpha^5]}{-4\beta^6 - 24\beta^5 - 180\beta^4 - 180\beta^3 + 9\beta^5\alpha + 45\beta^4\alpha - 10\beta^3\alpha^3 + 180\beta^3\alpha - 30\beta^2\alpha^3 + 9\beta\alpha^5 - 4\alpha^6 + 9\alpha^5}\right\}^{1/3} \tag{16}$$

In Table 2, values of the different correction factors $g(\epsilon)$ are presented for different bed porosities together with experimental data. It can be seen that the correction based on the Kuwabara flow model, $g(\epsilon)_K$, gives values comparable with those of Pfeffer's, $g(\epsilon)_P$, over the entire range of porosities. The correction based on the Neale and Nader flow field $g(\epsilon)_N$ gives close values to the statistical model (Sirkar solution) $g(\epsilon)_S$ for high values of the porosity ($\epsilon > 0.5$) and to the Happel model (Pfeffer's solution) $g(\epsilon)_P$ for low values of the porosity ($\epsilon < 0.5$). Comparing his and Pfeffer's theoretical results to experimental values of Thoenes and Kramers (1958) and Karabellas et al (1971), Sirkar found a similar tendency (1974).

The advantages of the present approach based on the Neale and Nader flow model are that no artificial boundary condition is introduced into the mathematics of the problem and that the results are applicable for the entire range of porosities. The restrictions of the model are $Re < 10$ and $Pe > 1\,000$. By comparing the experimental values, as obtained by Wilson and Geankoplis (1966) for a dilute bed of active and inactive spheres with $g(\epsilon)_N$, a very good agreement can be seen.

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TABLE 1. VALUES OF COEFFICIENT α AS A FUNCTION OF POROSITY ϵ

ϵ	α
1.0	0.0
0.99	0.2584
0.9	1.185
0.8	2.247
0.6	5.986
0.4	17.83
0.3	35.44
0.2	83.87

TABLE 2. VALUES OF CORRECTION FACTOR $g(\epsilon)$

		$g(\epsilon)$			
ϵ	Experimental	Pfeffer (Happel)	Present (Kuwabara)	Present (Neale and Nader)	Sirkar (Tam)
0.7	1.567**	2.00	2.08	1.87	1.81
0.6	1.83**	2.34	2.43	2.19	2.19
0.5	2.18**	2.78	2.86	2.62	2.79
0.476	3.06*	2.91	2.99	2.77	3.01
0.45	2.43**	3.045	3.12	2.92	—
0.40	2.73**	3.36	3.44	3.28	4.17
0.30	—	4.22	4.30	4.34	Not valid
0.26	4.59†	4.71	4.80	4.94	Not valid
0.25	—	4.85	4.92	5.13	Not valid

* Thoenes and Kramers $Sh = 3.05\, Pe^{1/3}$ } regular packing of spheres.
† Karabellas et al. $Sh = 4.58\, Pe^{1/3}$ }
** Wilson and Geankoplis $Sh = \frac{1.09}{\epsilon} Pe^{1/3}$.

NOTATION

a = sphere radius
b = cell envelope radius
A, B, C, F, H, G = constants in Equation (14)
D = diffusion coefficient
E = single sphere filtration efficiency
f, J = constants in Equation (16)
$g(\epsilon)$, $g(\epsilon)_K$, $g(\epsilon)_N$, $g(\epsilon)_P$, $g(\epsilon)_S$ = correction factors defined by Equation (8) for the case of the Kuwabara, Neale and Nader, Happel, and Tam flow models, respectively
K = Darcy's law constant
l = dimensionless cell radius, $l = b/a$
N = dimensionless concentration
Pe = Peclet number, $Pe = 2aU_o/D$
R = dimensionless radial coordinate
Re = Reynolds number, $Re = 2aU_o/\nu$
Sh = Sherwood number defined by Equation (7)
U_o = fluid superficial velocity
U_r, V_θ = dimensionless velocity components in the radial and angular direction, respectively
y = dimensionless coordinate

Greek Letters

α, β = constants in Equation (14)
ψ, $\bar{\psi}$ = stream functions
θ = angular coordinate
ϵ = bed porosity
Δ^2 = Laplace operator in spherical coordinates

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An Improvement of the Simple Model for Rotary Flow Cyclones

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A simplified model for the performance of a rotary flow cyclone has been reported by Ciliberti (1976) which has several inherent assumptions in it. A similar analysis is presented here that is somewhat more general, yet leading to expressions which are no more cumbersome and are in slightly better agreement with experimental observations.

DISCUSSION

The previous analysis of particle motion in the case of a rotary flow cyclone was based on the equation of drag force on a particle of a given size to the centrifugal force experienced by the particle. In the expression for the drag force, the inward radial gas velocity was neglected, giving rise to the equation

$$3\pi\mu d\left(\frac{dr}{dt}\right) = \frac{\rho_p \pi d^3}{6}(r\omega^2) \tag{1}$$

The determination of the angular velocity was based on an angular momentum balance about a cylindrical section of the core in which only the flux of momentum across the cylindrical area was accounted for, and the flux through the bottom and top were neglected. The following analysis attempts to include these factors.

A force balance on the particle shown in Figure 1 leads to the following expression:

$$\text{drag force} = \text{centrifugal force}$$

$$3\pi\mu d\left(\frac{dr}{dt} - V_r\right) = \frac{\pi d^3 \rho_p}{6}(r\omega^2) \tag{2}$$

This assumes Stokes Law drag forces, negligible acceleration in the radial direction, spherical, nonagglomerating particles, and that the tangential gas and particle velocities are equal. To solve this equation, V_r and ω must be determined.

The angular velocity may be obtained by looking at a section of the core which is assumed to be in solid body rotation. Figure 2 indicates the fluxes of angular momentum across this section's boundaries and leads to the following angular momentum balance:

$$\int_z^{z+\Delta z} \omega R_o \cdot R_o \cdot 2\pi R_o \rho V_r(R_o)\, dz + \int_o^{R_o} \omega r \cdot r \cdot 2\pi r \rho V_z dr \Big|_z = \int_o^{R_o} \omega r \cdot r \cdot 2\pi r \rho V_z dr \Big|_{z-\Delta z} \tag{3}$$

Performing the indicated integrations over r by assuming a flat V_z profile, and taking the limit as $\Delta z \to 0$, we obtain

$$\frac{d}{dz}(\omega V_z) = \frac{4V_r(R_o)\omega}{R_o} \tag{4}$$

The further assumption that the secondary gas flow enters the core uniformly leads to the expressions for V_z and $V_r(R_o)$:

$$V_r(R_o) = \frac{-Q_s}{2\pi R_o H} \tag{5}$$

$$V_z(z) = \frac{Q_p + \left(\frac{z}{H}\right) Q_s}{\pi R_o^2} \tag{6}$$

Substitution of these expressions into Equation (4) leads to this expression for $\omega(z)$:

$$\omega(z) = \omega(o)\left[1 + \left(\frac{z}{H}\right)\frac{Q_s}{Q_p}\right] \tag{7}$$

To determine the initial value of ω, some assumptions must be made about the design of the cyclone. It would be possible, for example, to evaluate ω if the value of S for the cyclone were known. Various values of S have